Long term prediction of marble erosion for the conservation of the statue of David of Michelangelo

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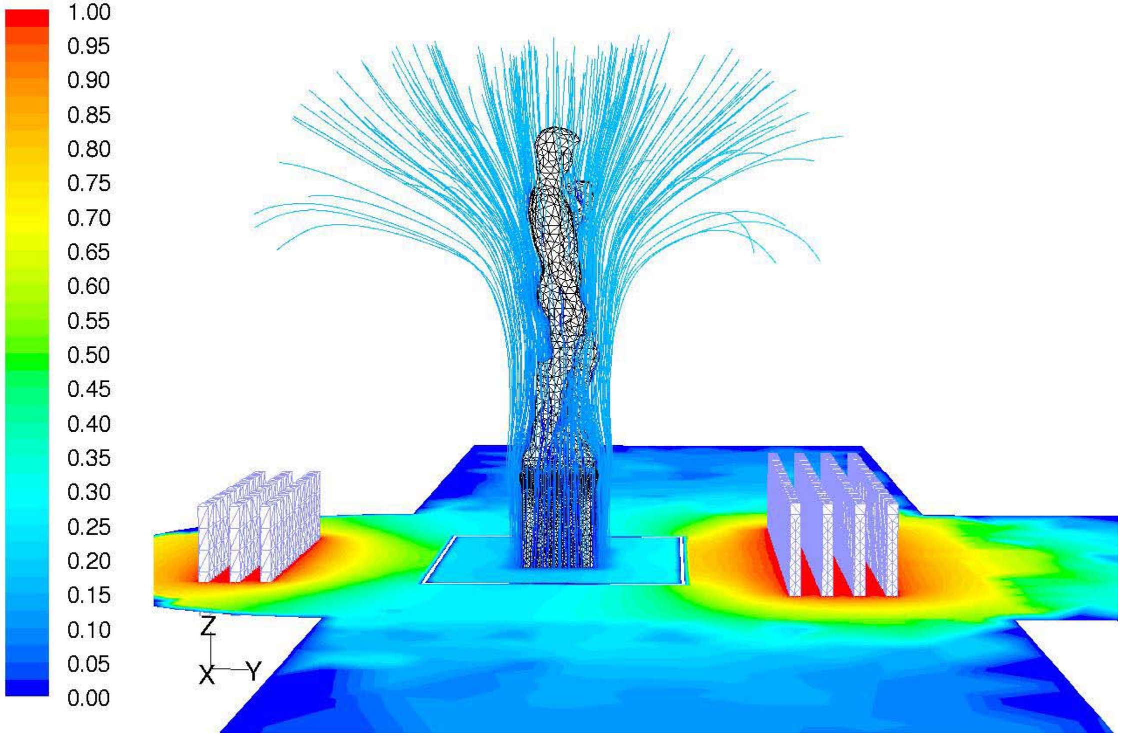
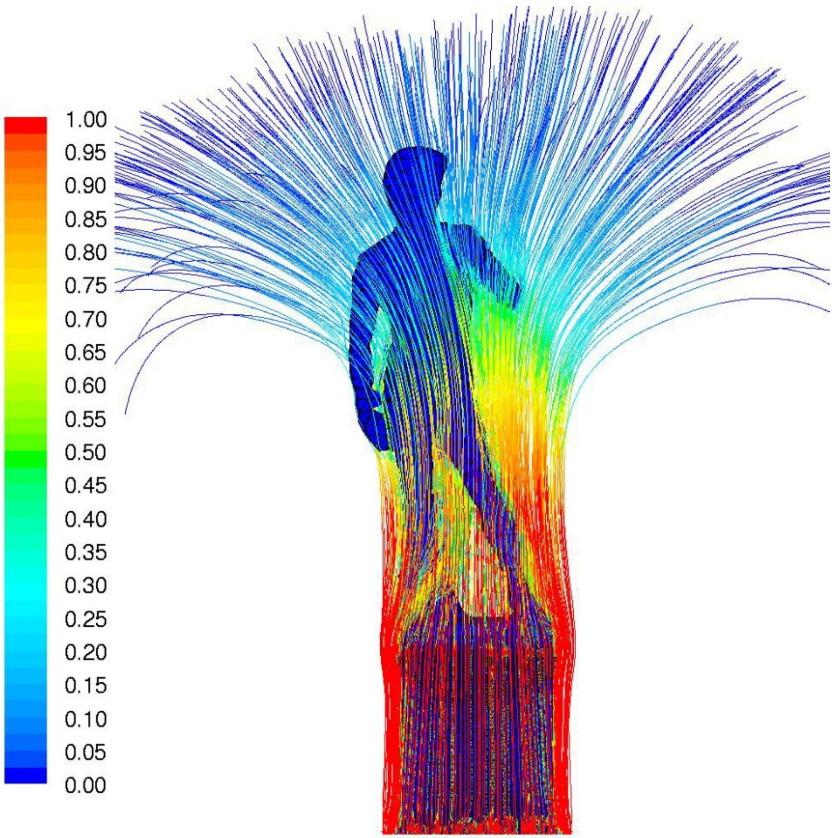
- Visitors represents a major vector of air pollutants
- The statue need protection from the pollutants deposition
- The current fruition condition can't be modified because of it's historic significance



- A research have been focusing on the implementation of a kind of air ventilation system in order to separate the statue from the surrounding pollutants
- A CFD tool was used in order to optimize the design of the ventilation system



The simulation result stated that the intake from the bottom of the statue (1m/s), with controlled RH and high efficient filtration, can efficiently separate the statue from the surrounding environment

- **The Management of the Museum suggested further investigation concerning long term erosion phenomena**

THE EROSION RISK MANAGEMENT

- Tulsa Model
- CFD discrete phase transport

The shape and hardness coefficient

$C(d_p)$ depends on the shape and on the hardness of the impinged surface.

$$ER = \sum_{p=1}^{N_{particle}} \frac{\dot{m}_p \, C(d_p) \, f(\alpha) \cdot v^{b(v)}}{A_{face}}$$

The experimental value of b is 1.73.

$$C(d_p) = 470.22 \cdot B^{-0.59} \cdot F_s$$

Symbol	Description	Unit
$\dot{m}_p$	Mass flow rate	kg/s
B	Brinell hardness	
F_s	Shape coefficient	*Dimentionless*
α	Impinging angle	*Sessagesimal*
v	Air velocity	m/s

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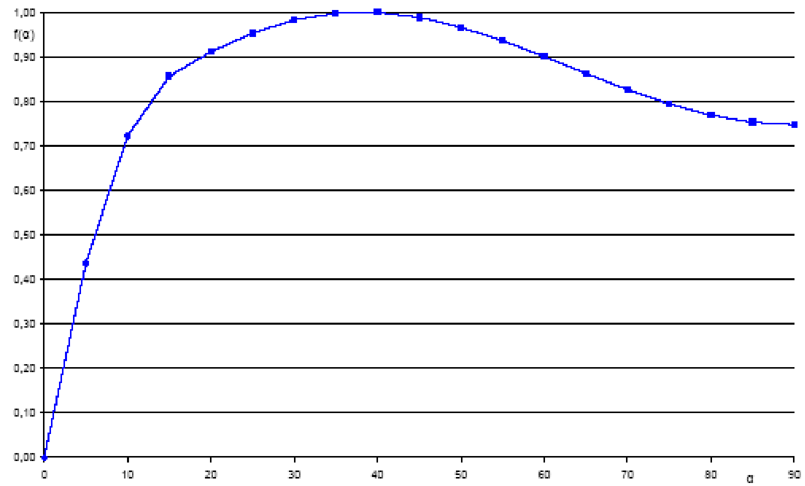
The normalized angle function f(α)

This funtion was sperimentally found for metal brittle materials with dry surfaces for a discrete phase dispersed in a gas.

$$ER = \sum_{p=1}^{N_{particle}} \frac{\dot{m}_p \cdot C(d_p) \cdot f(\alpha) \cdot v^{b(v)}}{A_{face}}$$

$$f(\alpha) = \frac{-0.384 \cdot \vartheta^2 + 0.227 \cdot \vartheta}{0.039} \qquad \vartheta \leq 15^\circ$$

$$f(\alpha) = \frac{0.03147 \cdot \cos^2 \vartheta \cdot sen\vartheta + 0.003609 \cdot sen^2 \vartheta}{0.039} \qquad \vartheta > 15^\circ$$



The Tulsa model considers a sand-metal erosion

- The Brinell hardness, which appears in the expression of ER, regarding only metal materials, makes to find a trustworthy value for Marble very difficult. Indeed hardness of stony material is given only by Mosh or more recently by schelometer scale.
- Neverthless an estimation of Mosh hardness of Marble was developed finding a material which appears in the two scales at the same time, with the same Mosh value of Marble.
- Having Marble the same Mosh hardness of gold, it makes possible to use a 34 Brinnell hardness for the stone specimen of the simulation

The Mosh hardness of Marble is between 2.5 and 3 like as Gold

Hardness	Mineral
1	Talc ($Mg_3Si_4O10(OH)_2$)
2	Gypsum ($CaSO_4 \cdot 2H_2O$)
3	Calcite ($CaCO_3$)
4	Fluorite (CaF_2)
5	Apatite ($Ca_5(PO_4)_3(OH^-,Cl^-,F^-)$)
6	Orthoclase Feldspar ($KAlSi_3O_8$)
7	Quartz (SiO_2)
8	Topaz ($Al_2SiO_4(OH^-,F^-)_2$)
9	Corundum (Al_2O_3)
10	Diamond (C)

Other material hardness (mosh) [27]	
2.5	Fingernail
2.5-3	**Gold, Silver**
3	Copper penny
4-4.5	Platinum
4-5	Iron
5.5	Knife blade
6-7	Glass
6.5	Iron pyrite
7+	Hardened steel file

Mosh scale for Stone Materials

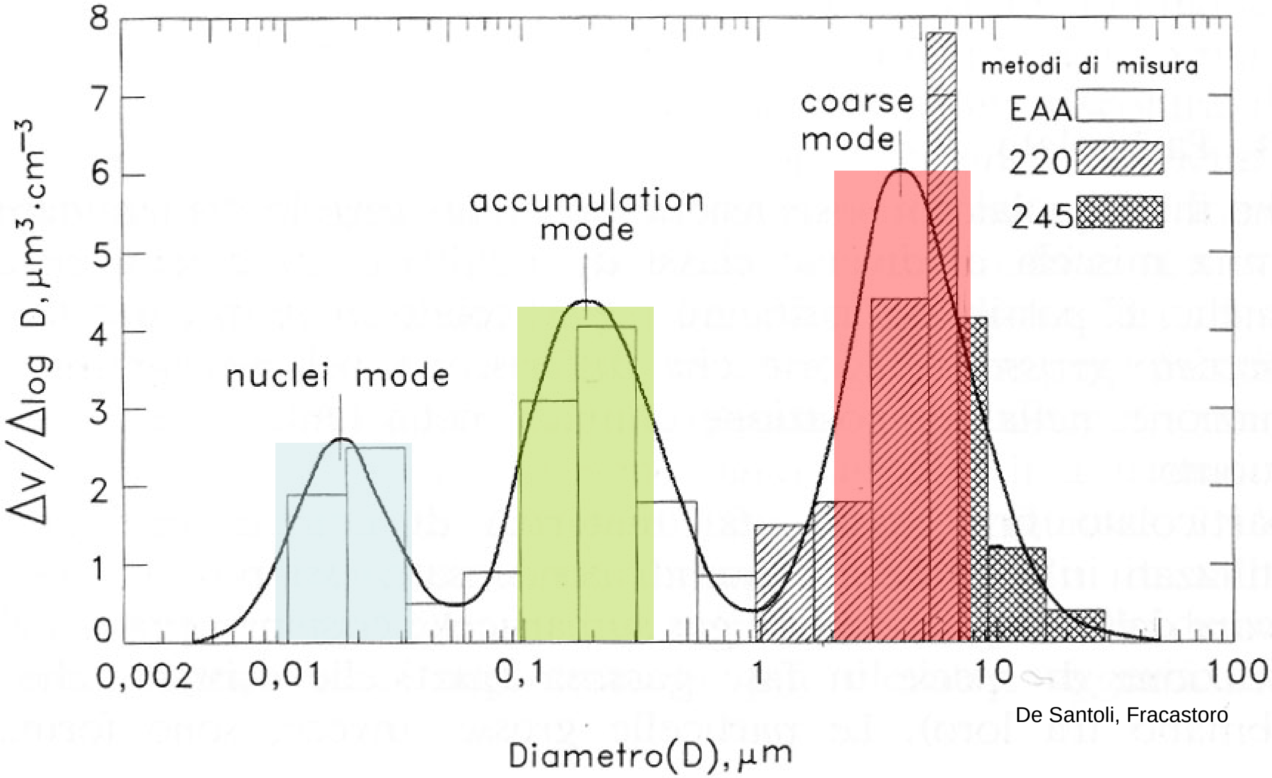
A 34 Brinnel hardness is estimated for marble.

5 DIFFERENT CONCENTRATIONS OF POLLUTANTS →

10000 μg/m³
1000 μg/m³
500 μg/m³
250 μg/m³
100 μg/m³

THE DIAMETER DISTRIBUTION OF PARTICLES

15% PM 0.5 35% PM 2.5 50% PM 10



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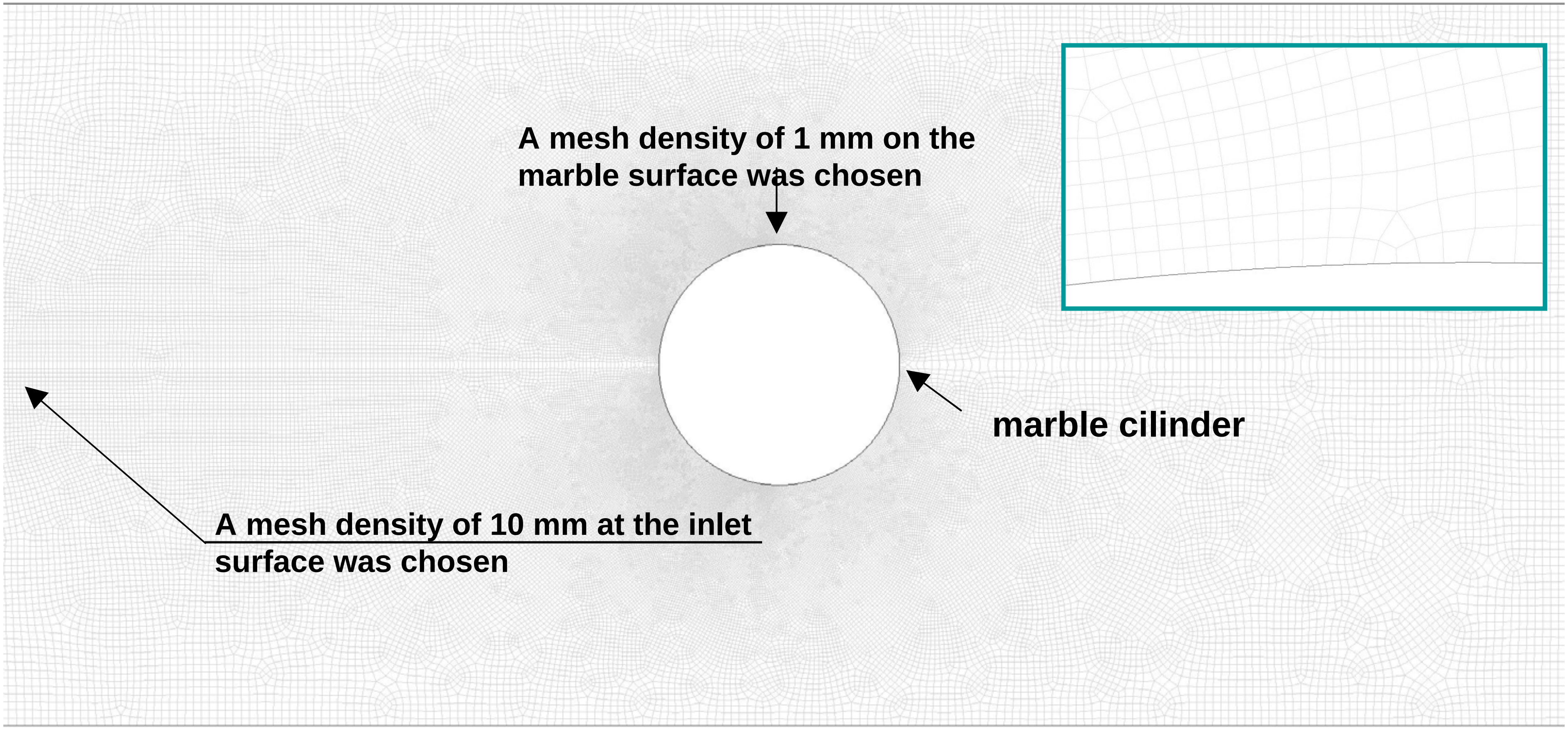
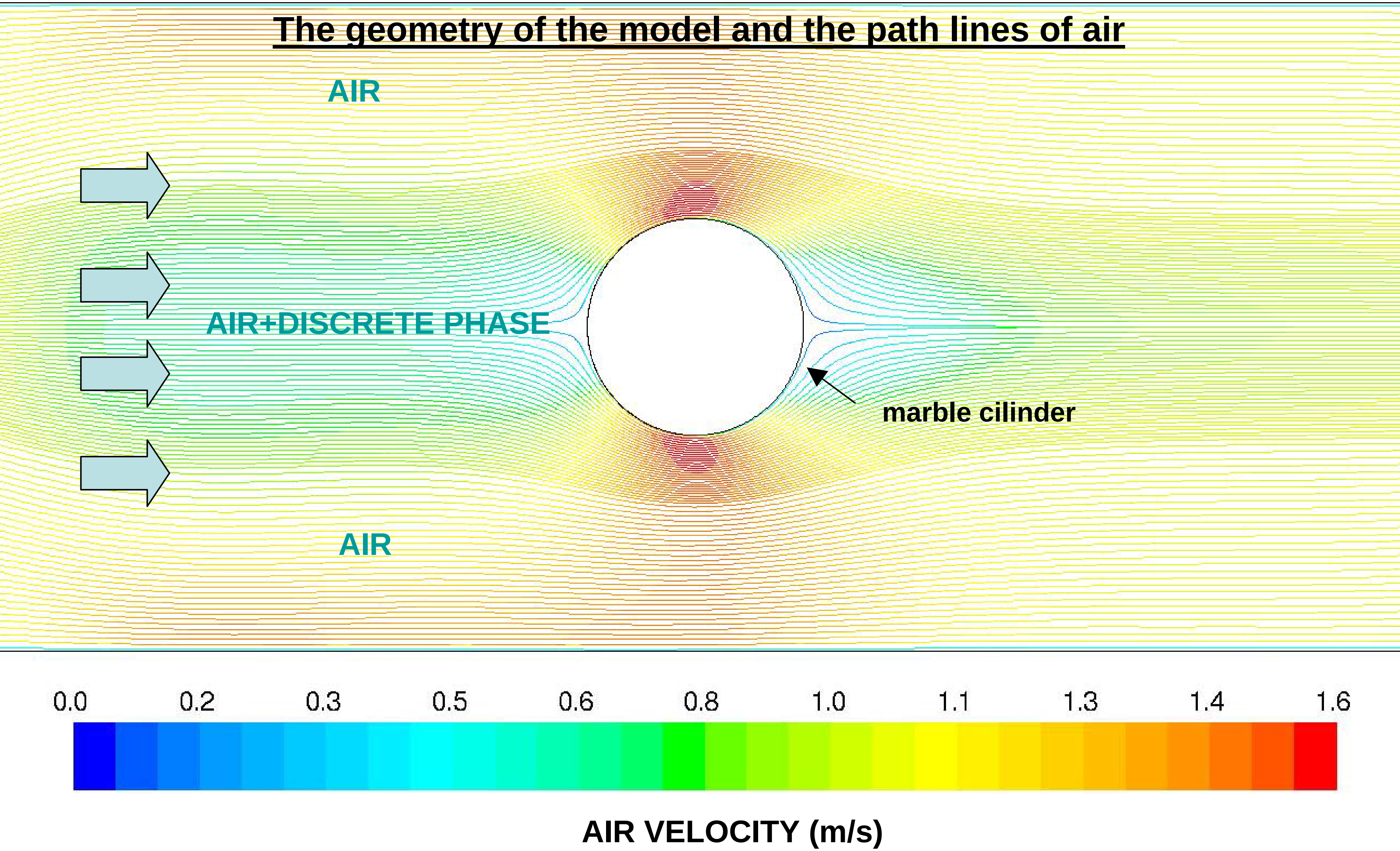
Defining the Mesh

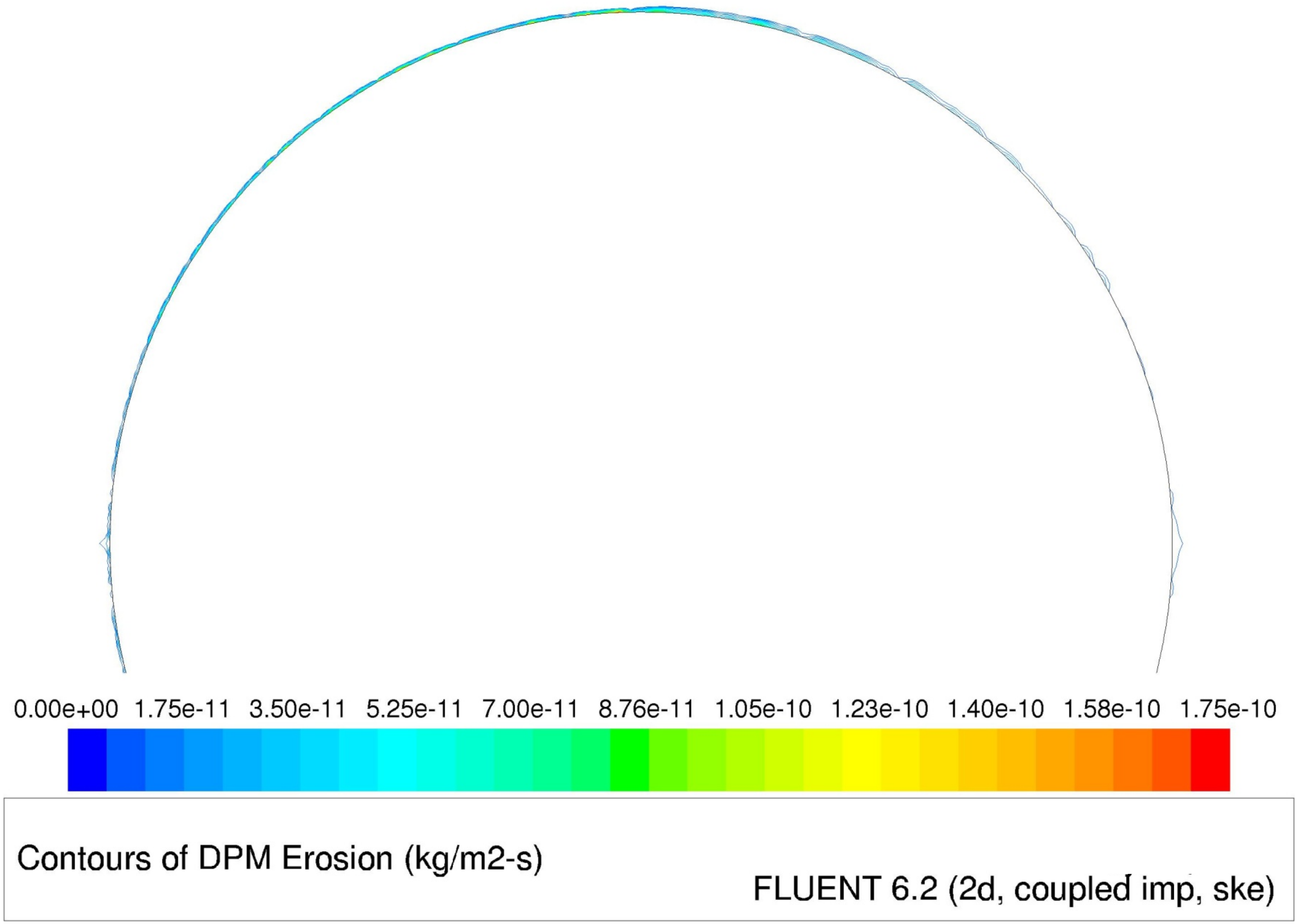


A variable density mesh was used in order to better assess the amount of material loss due to erosion.

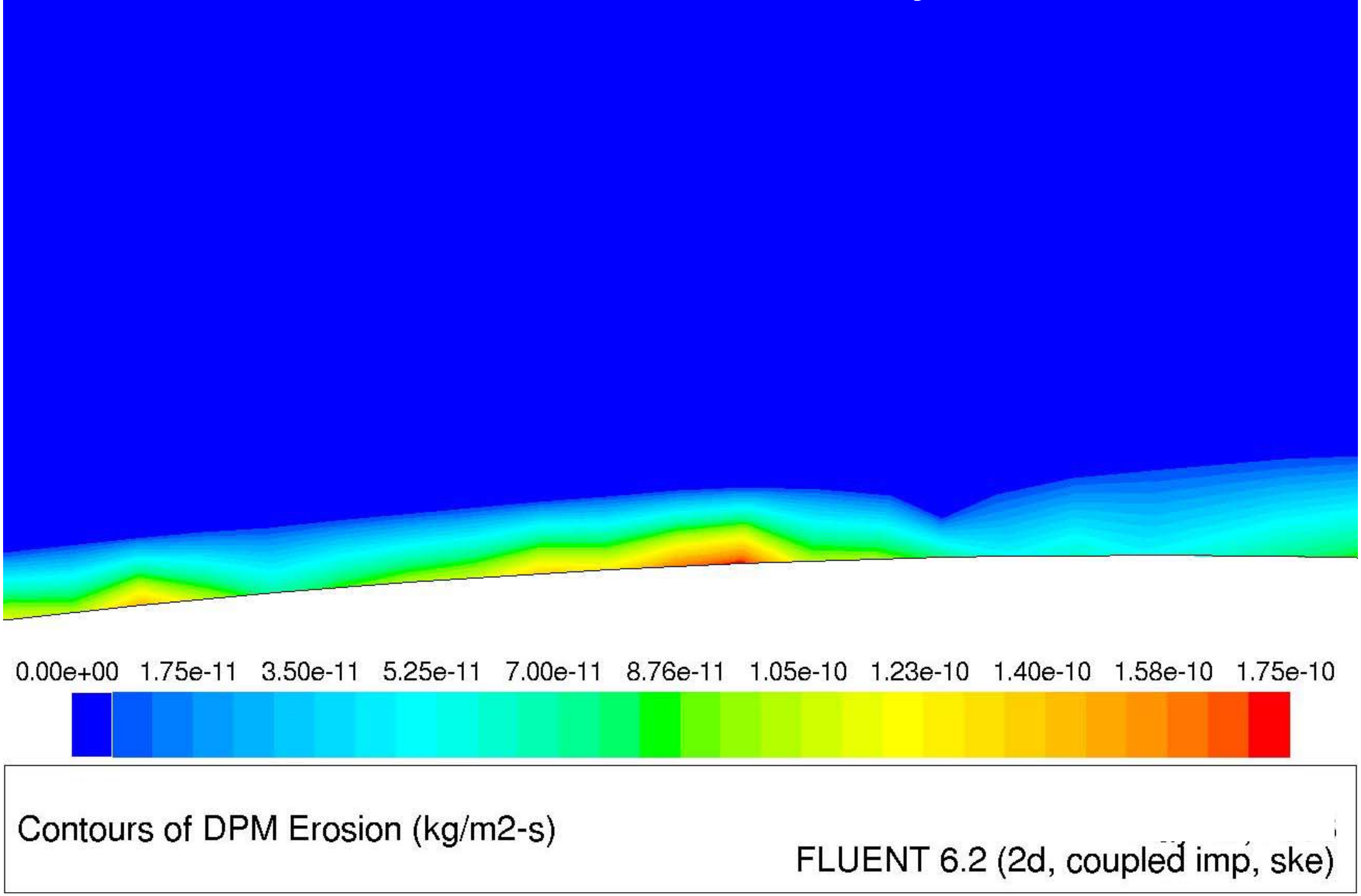
The geometry of the model and the path lines of air



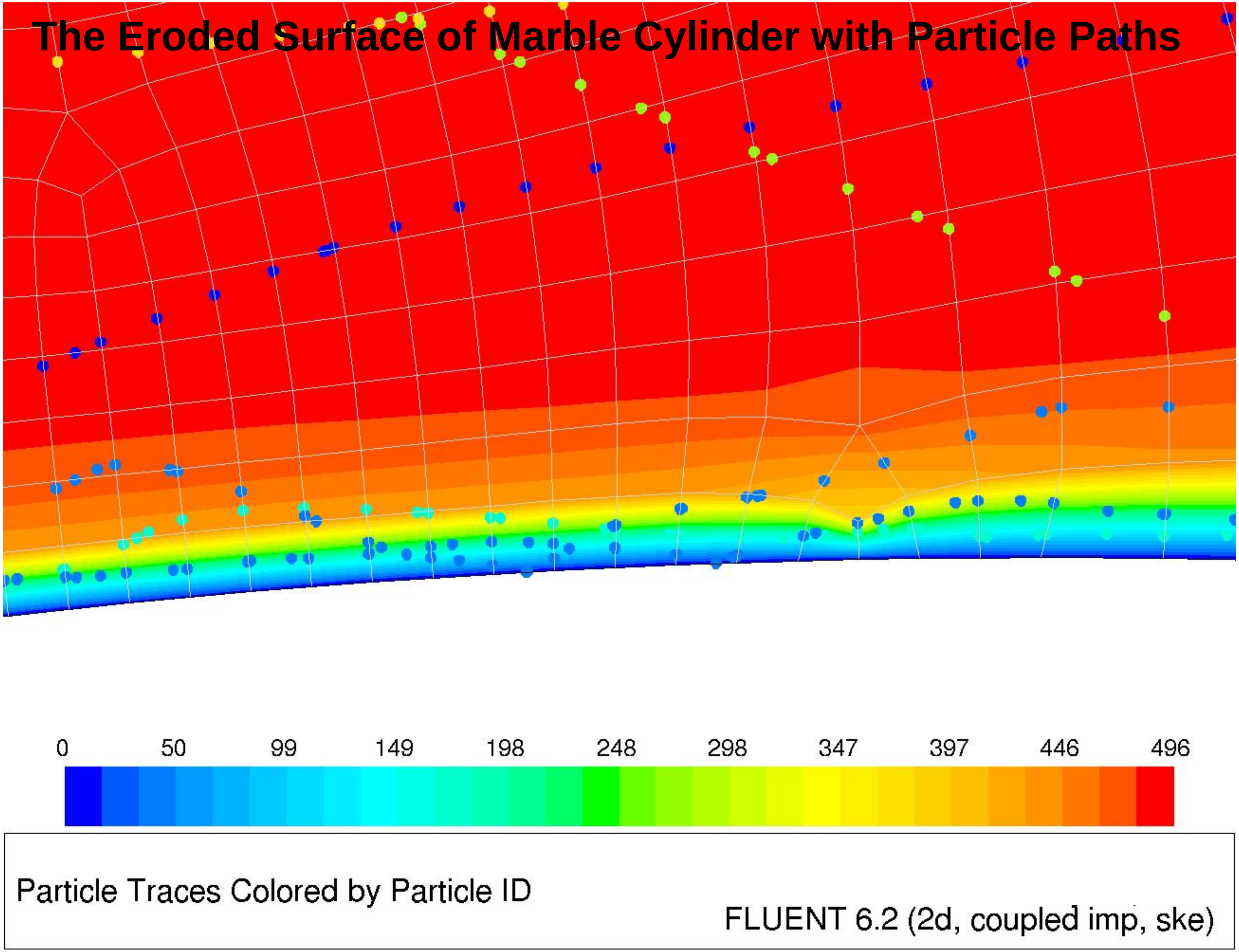
The result of simulations



The eroded surface of marble Cylinder



The Eroded Surface of Marble Cylinder with Particle Paths



RESULTS

Particle concentration ($\mu g/m^3$)	Particle flow rate (Kg/s)	ER average on front surface (kg/m^2s)	Erosion thickness after 100 yr (μm)
10000	$1.500\ 10^{-6}$	$3.901\ 10^{-10}$	512
1000	$1.500\ 10^{-7}$	$4.851\ 10^{-11}$	64
500	$0.750\ 10^{-7}$	$2.819\ 10^{-11}$	37
250	$0.375\ 10^{-7}$	$1.358\ 10^{-11}$	18
100	$1.500\ 10^{-8}$	$6.040\ 10^{-12}$	8

Conclusions

- The results, obtained using a very conservative model, show that the decay effect on marble surfaces could be controlled and considered negligible.
- The hypothesis of absence of a filtering section for a elapsed time more than tree times long if compared with the life cycle of the designed air curtain protection system of the statue of David, supports this statement with an even greater level of certainty.